#### FST 3-7 Notes

Topic: Composition of Functions

GOAL:

Formalize the concept of composition of functions by defining composition and introducing the o symbol.

## **SPUR Objectives**

A Find equations for and values of composites of functions.

F Identify properties of composites of functions.

# Vocabulary

composite

function composition

## **Definition of Composite Function**

Suppose f and g are functions. The composite of g with f, written  $g \circ f$ , is the function defined by

$$(g \circ f)(x) = g(f(x)).$$

The domain of  $g \circ f$  is the set of values of x in the domain of f for which f(x) is in the domain of g.

\* Composition of Functions is not commutative (order matters!)

**Example 1**: Let 
$$f(x) = x^2$$
 and  $g(x) = \frac{1}{3x+1}$ . Evaluate.

$$f\left(\frac{1}{3(4)+1}\right) = \frac{1}{13}$$

### On your own:

Let  $f(x) = 3x^2 - 3x$  and g(x) = x + 7. Evaluate:

a) 
$$(f \cdot g)(3) = f(g(3))$$
  
 $f(3+7) = f(10)$   
 $f(10) = 3(10)^2 - 3(10)$   
 $= 300 - 30$   
 $f(g(3)) = 270$ 

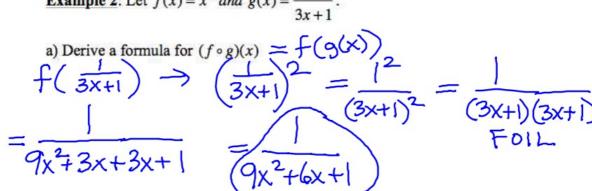
$$g(3(3)^{2}-3(3))$$

$$g(27-9) = g(18)$$

$$g(18) = 18+7$$

$$g(f(3)) = (25)$$

Example 2: Let  $f(x) = x^2$  and  $g(x) = \frac{1}{3x+1}$ .



b) Give a simplified formula for  $(g \circ f)(x) = g(f(x))$ 

c) Verify that  $f \circ g \neq g \circ f$  by graphing.

$$\frac{1}{(3x+1)^2} \neq \frac{1}{(3x^2+1)}$$

**Example 3:** Let 
$$f(x) = x^2$$
 and  $g(x) = \frac{1}{3x+1}$ . Find the domain of  $f \circ g$ .

To be in the domain of  $f \circ g$ , a number x must be in the domain of g, and the corresponding g(x) value must be in the domain of f.

Step 1: Find the domain of the "inside" (input) function. If there are any restrictions on the domain, **keep them**.

Step 2: Construct the composite function. Find the domain of this new function. If there are any restrictions on this domain, add them to the restrictions from Step 1. If there is an overlap, use the more restrictive domain.

Find the domain of  $f \circ g$ . f(g(x))(1) Find domain of inside function  $g(x) = \frac{1}{3x+1}$ with fractions, denominator  $\neq 0$   $3x+1\neq 0$  -1 - 1  $3x\neq -1$   $x\neq -1/3$ (2) Calculate f(g(x))  $f(\frac{1}{3x+1}) \Rightarrow (\frac{1}{3x+1})^2 = \frac{1}{(3x+1)^2} = \frac{1}{9x^2+6x+1}$ Find domain of  $9x^2+6x+1\neq 0$   $9x^2+6x+1\neq 0$   $3x+1)(3x+1)\neq 0$   $3x+1\neq 0$ 

**Example 4:** Find  $f \circ g$  and  $g \circ f$  and the domain of each.

$$f(x) = \underbrace{\frac{3x}{x-1}} \qquad g(x) = \frac{2}{x}$$

Find 
$$f \circ g$$
. State the domain.

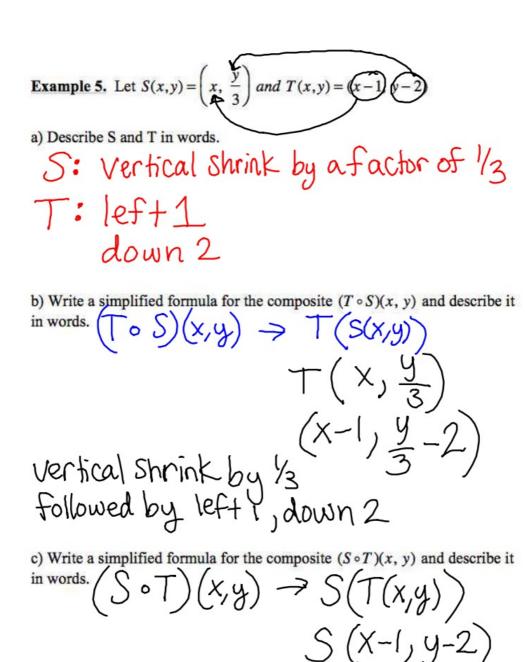
1)  $f \circ g = f(g(x))$  Domain of  $g(x) = \frac{2}{x}$   $x \neq 0$ 

2) Find  $f(g(x))$   $\frac{3(\frac{2}{x})}{f(\frac{2}{x})-1} = \frac{6}{2x} = \frac{6}{x}$ 

2)  $\frac{6}{x} \cdot \frac{x}{2-x} = \frac{6}{(\frac{2}{x})-1} = \frac{6}{2x} = \frac{6}{x}$ 

2)  $\frac{2}{x} - \frac{x}{x} = \frac{6}{x} = \frac{6}{x}$ 

2)  $\frac{2}{x} - \frac{x}{x} = \frac{6}{x$ 



Translation left 1, down 2 followed by vertical shrink of 1/3